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**UTILITY PATENT APPLICATION TRANSMITTAL
(Large Entity)***(Only for new nonprovisional applications under 37 CFR 1.53(b))*Docket No.
APP 1245-USTotal Pages in this Submission
3**TO THE ASSISTANT COMMISSIONER FOR PATENTS**Box Patent Application
Washington, D.C. 20231

Transmitted herewith for filing under 35 U.S.C. 111(a) and 37 C.F.R. 1.53(b) is a new utility patent application for an invention entitled:

A Method of Using Transient Faults to Verify the Security of a Cryptosystem

and invented by:

Dan Boneh - Richard A. DeMillo - Richard LiptonIf a **CONTINUATION APPLICATION**, check appropriate box and supply the requisite information:☒ **Continuation** ☐ **Divisional** ☐ **Continuation-in-part (CIP)** of prior application No.: 08/933,541

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Enclosed are:

Application Elements

1. ☒ Filing fee as calculated and transmitted as described below
2. ☒ Specification having 53 pages and including the following:
 - a. ☒ Descriptive Title of the Invention
 - b. ☐ Cross References to Related Applications *(if applicable)*
 - c. ☐ Statement Regarding Federally-sponsored Research/Development *(if applicable)*
 - d. ☐ Reference to Microfiche Appendix *(if applicable)*
 - e. ☒ Background of the Invention
 - f. ☒ Brief Summary of the Invention
 - g. ☒ Brief Description of the Drawings *(if drawings filed)*
 - h. ☒ Detailed Description
 - i. ☒ Claim(s) as Classified Below
 - j. ☒ Abstract of the Disclosure

UTILITY PATENT APPLICATION TRANSMITTAL (Large Entity)

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Application Elements (Continued)

3. ☒ Drawing(s) (when necessary as prescribed by 35 USC 113)
- a. ☐ Formal Number of Sheets _____
- b. ☒ Informal Number of Sheets 8
4. ☐ Oath or Declaration
- a. ☐ Newly executed (original or copy) ☐ Unexecuted
- b. ☐ Copy from a prior application (37 CFR 1.63(d)) (for continuation/divisional application only)
- c. ☐ With Power of Attorney ☐ Without Power of Attorney
- d. ☐ DELETION OF INVENTOR(S)
Signed statement attached deleting inventor(s) named in the prior application,
see 37 C.F.R. 1.63(d)(2) and 1.33(b).
5. ☐ Incorporation By Reference (usable if Box 4b is checked)
The entire disclosure of the prior application, from which a copy of the oath or declaration is supplied under Box 4b, is considered as being part of the disclosure of the accompanying application and is hereby incorporated by reference therein.
6. ☐ Computer Program in Microfiche (Appendix)
7. ☐ Nucleotide and/or Amino Acid Sequence Submission (if applicable, all must be included)
- a. ☐ Paper Copy
- b. ☐ Computer Readable Copy (identical to computer copy)
- c. ☐ Statement Verifying Identical Paper and Computer Readable Copy

Accompanying Application Parts

8. ☐ Assignment Papers (cover sheet & document(s))
9. ☐ 37 CFR 3.73(B) Statement (when there is an assignee)
10. ☐ English Translation Document (if applicable)
11. ☐ Information Disclosure Statement/PTO-1449 ☐ Copies of IDS Citations
12. ☐ Preliminary Amendment
13. ☒ Acknowledgment postcard
14. ☒ Certificate of Mailing
- ☐ First Class ☒ Express Mail (Specify Label No.): EJ167603059US

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Accompanying Application Parts (Continued)

15. ☐ Certified Copy of Priority Document(s) (if foreign priority is claimed)

16. ☐ Additional Enclosures (please identify below):

Fee Calculation and Transmittal

CLAIMS AS FILED

For	#Filed	#Allowed	#Extra	Rate	Fee
Total Claims	39	- 20 =	19	x \$18.00	\$342.00
Indep. Claims	7	- 3 =	4	x \$78.00	\$312.00
Multiple Dependent Claims (check if applicable) <input type="checkbox"/>					\$0.00
BASIC FEE					\$690.00
OTHER FEE (specify purpose)					\$0.00
TOTAL FILING FEE					\$1,344.00

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Signature
Joseph Giordano, Reg. No.: 35,158
Telcordia Technologies, Inc.
445 South Street Room 1G-112R
Morristown, New Jersey 07960

Dated: MARCH 1, 2000

CC:

**A METHOD OF USING TRANSIENT FAULTS
TO VERIFY THE SECURITY OF A CRYPTOSYSTEM**

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BACKGROUND OF THE INVENTION

Field of the Invention

The present invention relates to cryptanalysis and, more particularly, relates to methods for "cracking", or deciphering, cryptosystems, by analyzing one or more erroneous outputs to infer information ordinarily difficult or impossible for a party not privy to secret information. Knowing how a cryptosystem may be cracked suggests methods for avoiding attacks on the cryptosystem, thus further improving the integrity of the cryptosystem. A security expert or cryptosystem designer may use the inventive methods in the design of cryptography devices to verify that an existing or proposed device is impervious to such attacks.

Discussion of Related Art

Cryptography has become essential to the acceptance of electronic commerce and sensitive electronic communications. For example, secure digital signatures and verification methods provide high assurance that a party is who it represents itself to be. This assurance is vital to the general acceptance of, for example, commerce over the Internet, the use of electronic money, cellular communications, and remote computer login procedures. Typically, certain well-known cryptographic methods are used to encrypt information in a manner that is very difficult to decrypt without certain secret information, thus making

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these signatures and verifications secure. One type of cryptographic method which is commonly used is public key cryptography.

1. Public Key Cryptography

In a typical public key cryptographic system, each party i has a public key (or exponent) P_i and a secret key (or exponent) S_i . The public key P_i is known to everyone, but the secret key S_i is known only to party i . A plain text message m to user i is encrypted to form the cipher text message x using a public operation P which makes use of the public key P_i known to everyone, i.e., $x = P(m, P_i)$. The cipher text message x is decrypted using a secret operation S which makes use of the secret key S_i , i.e., $m = S(x, S_i)$. Only party i who has the secret key S_i can perform the secret operation to decrypt the encrypted message x to obtain clear text message m .

Public key cryptographic techniques may be used for authentication. Authentication is a (theoretically) fool-proof technique for a party to verify that a party contacting it is the party it asserts to be. For example, a confidential network may require that a party authenticate itself before gaining access to the network.

If it is true that $P(S(x, S_i), P_i) = x$ (recall the $S(x, S_i) = m$, resulting in $P(m, P_i) = x$), then the owner of the corresponding keys P_i, S_i could sign message m by producing $E = S(m, S_i)$, where E indicates the signature. The verifier, given x and E , will verify $x = P(E, P_i)$. One type of a cryptography system could be used for verification as follows: challenge the party claiming to be i with message x and ask the party to sign the message x using his secret key S_i , then

verify the signature using P_i . More efficient and secure authentication protocols may be used, such as the Fiat-Shamir and Schnorr protocols discussed below.

Fig. 1A is a block diagram of a typical cryptography device 100. The device 100 has a processor 102 including one or more CPUs 102, a main memory 104, a disk memory 106, an input/output device 108, and a network interface 110. The devices 102-110 are connected to a bus 120 which transfers data, i.e., instructions and information between each of these devices 102-110.

Fig. 1B illustrates a network 150 over which cryptography devices 100 may communicate. Two or more cryptography devices 100, 100' may be connected to a communications network 152, such as a wide area network; which may be the Internet, a telephone network, or leased lines; or a local area network. Each device 100 may include a modem 154 or other network communication device to send encrypted messages over the communications network 152. A cryptography device 100 may be a gateway to a sub-network 156. That is, the device 100 may be an interface between a wide area network 152 and a local area (sub) network 156.

An example of a public key cryptographic technique which may be performed by the device 100 is the well known RSA technique. In accordance with this technique, a party i has stored in memory 104 or 106 its own public key (or exponent) e_i and modulus N (where N is a product of two large prime numbers p, q) and a secret key in the form of an exponent s_i . It has stored or otherwise obtained the public key e_j of a party to which it wishes to send a message. The party may have a plain text message m which it wishes to send

Party i 's public key is the square of these invertible elements (bits)
 $v_1 = s_1^2, \dots, v_t = s_t^2 \pmod{N}$. Party i authenticates itself to party j using the
 following protocol:

1. Party i 's cryptography device selects a random r , generates $r^2 \pmod{N}$, and transmits this value to party j 's cryptography device.
2. Party j 's cryptography device selects a random subset $S \subseteq \{1, \dots, t\}$, and transmits the subset to party i via an I/O.
3. Party i 's cryptography device computes $y = r \prod_{i \in S} s_i \pmod{N}$ and transmits y to party j .
4. Party j 's device verifies party i 's identity by checking that $y^2 = r^2 \prod_{i \in S} v_i \pmod{N}$.

The Schnorr authentication scheme is another cryptosystem for a first party to authentic its identity to a second party. The security of the Schnorr authentication scheme is based on the difficulty of computing discrete log modulo a prime. In Schnorr's authentication scheme, party i and party j agree on a prime number p and a generator g of Z_p^* , where Z_p^* is group of integers modulo p and relatively prime to p . Party i chooses a secret integer s_i and publishes $y_i = g^{s_i} \pmod{p}$ as party i 's public key. Party i authenticates itself to party j by engaging in the following protocol:

1. Party i 's cryptography device selects a random integer $r \in [0, p)$ and sends $z = g^r \pmod{p}$ to party j 's cryptography device via an I/O

2. Party j 's cryptography device selects a random integer $t \in [0, T]$ and sends t to party i via an I/O. Here, $T < p$ is an upper bound chosen beforehand.
3. Party j 's device sends $u = r + ts \bmod p-1$ to party j 's device.
4. Party j 's device verifies that $g^u = zy^t \bmod p$.

Cryptography schemes such as Schnorr have the property that if two distinct messages are signed using the same random element (e.g., r), then the secret key of the signer can be computed by anyone having the messages, the signatures, and public information such as the public key of the signer.

2. Prior Art Difficulties Cracking Cryptosystems

Cracking the RSA public key cryptosystem, and several other cryptosystems, is difficult because it typically requires that the modulus be factored (or other operation of comparable complexity). This is particularly difficult. It takes thousands of hours of computing time to factor a 512 bit modulus. RSA currently uses a 512 bit modulus, but it is expected that this may be upgraded in the future to a 1024 bit modulus. However, if the modulus may be determined without significant factoring, the computing time may be greatly reduced and the security of the cryptosystem compromised.

In an article "Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems," Proc. of Crypto '96, P. Kocher proposes that a few bits of a modulus may be obtained by the amount of time certain operations took to be performed. This allowed the cryptosystem to be cracked without factoring. The drawbacks of this method are (1) it requires very precise timing

of the length of time taken to perform certain calculations; and (2) it requires a large number of samples.

3. Reasons For Cracking A Cryptosystem

The availability of electronic commerce and certain electronic communications depend on difficult-to-crack cryptosystems to prevent unauthorized access to the secured information. If, for example, an adversary obtains a party's secret key, the adversary could electronically forge the party's signature without the party's knowledge. As another example, the adversary could present itself to third parties as the party whose secret key was obtained. Moreover, once obtained, the secret key may be duplicated and shared with others. Thus, it is vitally important that the cryptosystem used to protect important information be difficult to crack.

A threat model for cracking a cryptosystem is useful because it verifies whether a cryptosystem or cryptography device is vulnerable to that attack. If so, the system or device is no longer considered to be secure. This is true because in the cryptography community, the mere possibility of an attack on a cryptosystem is universally accepted as very serious. Security experts must assume that the cryptosystem is no longer safe from adversaries. Thus, a method for cracking cryptosystems is an exceptionally useful tool for security experts and cryptosystem designers testing existing cryptosystems and developing new cryptosystems. The cracking method may be applied to an existing or a proposed system to verify that the system is impervious to the attack. Thus, the cracking method may also be used to design cryptosystems impervious to the attack.

Therefore, it is an object of the present invention to provide a method for cracking the public key signature cryptosystems without factoring the modulus.

It is another object of the present invention to provide a method for cracking cryptosystems using the Chinese Remainder Theorem.

5 It is yet a further object of the present invention to provide a method for cracking authentication cryptosystems.

It is yet another object of the present invention to use transient errors in encrypted data to determine secret information.

10 It is yet a further object of the present invention to provide methods for testing the security of a cryptosystem.

It is a further object of the present invention to provide a method for providing a cryptosystem and/or cryptography device impervious to cracking due to transient hardware faults.

15 **SUMMARY OF THE INVENTION**

The present invention is directed to methods for using one or more faulty computations made by a cryptography device to infer secret information stored in the cryptography device. The inventive method is based on the well-accepted proposition that no computing system is perfectly fault free. In a preferred method, a security expert or cryptosystem designer may intentionally induce a tamper proof device or other cryptography device to generate a faulty computation by subjecting the device, such as a smart card, to physical stress. Such physical stress may be, for example, certain types of radiation, atypical voltage levels, or a higher clock rate than the device was designed to operate

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at or accommodate. Cryptosystems and/or cryptography devices should preferably be impervious to the attacks described herein. If not, the system or device should desirably be modified. In some cases it may be desirable to discard the system.

5 In certain cryptosystems, such as a signature scheme based on the well known Chinese Remainder Theorem, a single error of any type is sufficient to crack the system. In certain other cryptosystems, such as certain authentication schemes, repeated errors of a specific type are used to crack the system. The inventive methods are useful tools for security experts and
10 cryptography experts when testing or developing a cryptosystem or cryptography device. Thus, the inventive method may be used to provide cryptosystems and/or cryptography devices impervious to cracking due to transient hardware faults.

In a first embodiment of the present invention, the RSA Chinese
15 Remainder Theorem based signature scheme and Rabin's Signature scheme (both of which may separate into linear components) are cracked by comparing a single erroneous signature on a message with a correct signature on the same message. In a second embodiment, these two schemes may be cracked with only a single erroneous signature if the content of the signed message is
20 known.

In a third embodiment, a certain type of fault called a register fault is used to crack the Fiat-Shamir and Schnorr authentication schemes. This is done by receiving a correct and a faulty value during an authentication process

to determine a secret value. Using this secret value, sets of data may be constructed which will reveal the other party's secret key.

In a fourth embodiment, erroneous signatures of randomly selected messages are each used to obtain a portion of a secret exponent. When a
5 sufficient number of bits are obtained, the remaining bits may be "guessed" to obtain the entire secret exponent.

The inventive method is a creative use of a cryptography device's miscalculations. Because it is believed that all computers are prone to error, even cryptosystem servers stored in a secure environment may not be secure
10 from these attacks. Thus, even such servers should be tested using the inventive method cracking cryptosystems. These attacks reveal an important finding: cryptography devices -- from smart cards to network servers used by certification authorities which oversee the distribution of public key certificates -
- should now not only conceal their inner circuitry (to avoid revealing its secret
15 key), but must also be fault resistant, to avoid generating erroneous calculations. The present invention provides a method for designing and implementing cryptosystems and cryptography devices impervious to cracking due to transient hardware faults.

20 **BRIEF DESCRIPTION OF THE DRAWINGS**

The present invention is described with reference to the following figures:

Fig. 1A is a block diagram of a typical cryptography device;

Fig. 1B illustrates a communications network over which cryptography devices may communicate;

Fig. 2 is a block diagram of a typical tamper proof device, such as a smart card.

Fig. 3 illustrates a first method according to the present invention;

Fig. 4 illustrates a second method according to the present invention;

Fig. 5 illustrates a third method according to the present invention;

5 Figs. 6A and 6B are flow charts illustrating two conventional exponentiation functions; and

Fig. 7 is a flow chart of an inventive method used with the methods of Figs. 6A and 6B.

10 **DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS**

The present invention is described in the following sections:

I. Types of Faults which may occur which permit certain cryptosystems to be cracked are described with reference to Fig. 2.

15 II. Cracking Cryptographic Signature Implementations that Use the Chinese Remainder Theorem is described with reference to Figs. 3 and 4, including discussion of the RSA Signature Scheme and the Chinese Remainder Theorem, Cracking the RSA Signature Scheme, and Cracking the Rabin Signature Scheme.

20 III. Using Register Faults To Break Cryptosystems is described with reference to Figs. 5, 6A, 6B, and 7, including discussion of Using Register Faults to Attack the Fiat-Shamir Authentication Scheme, Using Register Faults to Crack Schnorr's Authentication Scheme, and Using Register Faults to Crack Other RSA Implementations.

- IV. Providing Cryptosystems and Cryptography Devices which Resist Tampering Due to Hardware Faults is described.
- V. A Conclusion is provided.

5 I. **Types of Faults**

a. **Overview of Faults**

Several types of faults may enable a cryptosystem to be cracked. These faults include transient hardware faults, latent faults, and induced faults.

Cryptography devices, such as the device illustrated in Fig. 1A, described
10 above, are subject to random transient hardware faults. Random transient hardware faults may cause an erroneous output from the cryptography device. Referring to Fig. 1A, a random transient hardware fault in the processor 102 or memory 104, 106 may cause the certification authority to generate on rare occasion a faulty certificate. If a faulty certificate is sent to a client, that client
15 may be able to break a certification authority's system and generate fake certificates.

A latent fault is a hardware or software bug which may be difficult to detect. Such bugs may occur in the design of the processor 102, or in the design of software stored in the main memory 104 or the disk memory 106.
20 On rare occasions such bugs may cause a certification authority or other cryptography device to generate a faulty output.

Induced faults may occur when a security expert or cryptosystem designer has physical access to a cryptography device. The security expert or cryptosystem designer may purposely induce hardware faults by, for example,

attacking a tamper proof device by deliberately causing it to malfunction. An induced fault may, for example, briefly alter a value stored in the main memory 104 or the disk memory 106. Erroneous values computed by the device allow the security expert or cryptosystem designer to extract secret information stored in the cryptography device.

The present invention generally assumes that any faults generated by the cryptography device are transient. That is, the faults only affect current data, but not subsequent data. A transient fault may be a bit stored in a register which spontaneously flips or a gate which spontaneously produces an incorrect value. In such instances, the hardware system is typically unaware that any change has taken place. The present invention also assumes that the probability of such faults is so small that only a small number of them ever occur during a single computation.

b. Register Faults

A certain type of fault -- a register fault -- is used to crack certain public key authentication cryptosystems, as described below. A register fault is a transient corruption of data stored in one or more registers. Because one or a few bits in a register are corrupted, the erroneous calculation will have certain predictable properties (such as being a power of 2 or a sum of a few powers of 2).

As seen in Fig. 2, a tamper proof device 200, such as a smart card, comprises circuitry such as a processor 202 and a small amount of memory 204. The circuitry 202 performs certain arithmetic operations and the memory (typically several registers 206 and a small RAM 208) stores temporary values.

An I/O 210 is provided to receive and transmit data. An electrically erasable programmable read only memory (EEPROM) 212 may be provided for storing secret information, such as secret keys. These components 202 - 210 are connected by a bus 220.

5 With low probability, one or a few of the bits of the value stored in some register 206 may invert (e.g., change from a logic 0 to a logic 1 or vice-versa). It is assumed that this event occurs with sufficiently low probability so that there is some likelihood of a fault occurring only once throughout a computation. These errors may be transient and the hardware may not be
10 aware that the data corruption has occurred.

 Under normal operating conditions, hardware is substantially error free. However, when such hardware is placed under physical stress, such as being placed in an extreme environment such as exposing it to certain radiation, atypical voltage levels, or fast clock signals, errors are likely to occur. This
15 extreme environment may not affect the circuitry, but may cause certain register cells to spontaneously, temporarily invert. Such faults are referred to herein as "register faults". A security expert or cryptosystem designer may intentionally subject a tamper proof device 200, such as a smart card (or other cryptography device), to an extreme environment in order to test whether the device may be
20 induced to generate an erroneous output. If so, the system may be cracked. Also, it is a well-accepted that no computing system is entirely error-free. Thus, even in the absence of physical stress any cryptography device is susceptible to generating an erroneous output on rare occasions.

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II. Cracking Cryptographic Signature Implementations That Use the Chinese Remainder Theorem

5 One version of the present invention relies on the Chinese Remainder Theorem (CRT) to crack the RSA/CRT and Rabin modular square schemes. The Chinese Remainder Theorem is well known and described, for example, in A. Aho, J. Hopcroft, and J. Ullman, The Design and Analysis of Computer Algorithms, pp. 294-303 (Addison-Wesley 1974). The content of this
10 reference is incorporated herein by reference.

a. The RSA Signature Scheme And The Chinese Remainder Theorem

 The RSA signature scheme may be implemented in a tamper proof device 200, such as a smart card, and may be used to perform various encryption and decryption functions for its owner, party i . The tamper proof device typically
15 contains in the registers 206 a secret RSA decryption key which is used to decrypt messages for party i . This device 200 may be used, for example, to prepare digital signatures for party i , to authenticate party i to another party j , and to decrypt incoming encrypted messages. Assume that some secret information (such as party i 's secret key) is stored in a tamper proof device.
20 Because the device 200 is tamper proof, it cannot be opened and its contents examined. Thus, it is assumed that the secret information stored in the device cannot be extracted by opening the device.

 For illustrative purposes, the present version of the invention will be described as a device for obtaining digital signatures for party i . Let $N=pq$ be
25 a product of two large prime numbers. To sign a message m using the RSA signature scheme, the tamper proof device 200 uses the processor 202 to

compute $E = m^{s_i} \pmod{N}$ where s_i is a secret exponent stored in the register 206. The message m is assumed to be an integer in the range from 1 to N . As described above, the security of this system relies on the fact that factoring the modulus N is difficult. If the factors p, q of N are known, one can easily crack the system and sign documents without prior knowledge of the secret exponent s_i .

The computationally expensive part of signing using the RSA scheme is the modular exponentiation of the input m , which is performed by the processor 202. For efficiency, many implementations of the RSA scheme exponentiate signature E into two portions E_1 and E_2 as follows: first $E_1 = x^s \pmod{p}$ and $E_2 = x^s \pmod{q}$ are computed. Second, the Chinese Remainder Theorem is used to compute the RSA scheme signature $E = m^{s_i} \pmod{N}$.

Let a, b be two integers pre-computed by the processor 202 and stored in memory 206, which integers satisfy:

$$\begin{array}{ll} a \equiv 1 \pmod{p} & b \equiv 0 \pmod{p} \\ a \equiv 0 \pmod{q} & b \equiv 1 \pmod{q} \end{array} \quad \text{and} \quad (2)$$

Such integers always exists and can easily be determined by the processor 202 given p and q . It now follows that:

$$E = aE_1 + bE_2 \pmod{N} \quad (3)$$

The signature E is computed by processor 202 by forming a linear combination of E_1, E_2 . This exponentiation algorithm is more efficient than using the

processor 202 repeatedly to square modulo N because the numbers involved are smaller.

b. Cracking the RSA Signature Scheme

5 **1. Cracking the RSA Signature Scheme By
Comparing a Correct and an Incorrect Signature**

Using the linear combination set out above, the modulus N may be determined by a cryptography device such as the device 100 seen in Fig. 1A or a computer or other processor by comparing a correct signature E with an
10 incorrect signature \hat{E} for the same message. The inventive method is illustrated in Fig. 3. Let m be a plain text message and let $E = m^{si} \pmod{N}$ be the correct cryptographic signature of the message received by cryptographic device 100 at the I/O 108 and stored in memory 104 or 106. Let \hat{E} be a faulty
15 cryptographic signature for the same message m , and which is also received by the device 100 and stored in memory 104 or 106.

As seen in Fig. 3, party i 's cryptography device 200 generates signature E for message m . E is transmitted to party j 's cryptography device (or other processor) 100. Party j 's cryptography device stores E in memory (step 1).
20 Party j may be a security expert or cryptosystem designer. For clarity of illustration, the inventive method is described as a first cryptography device generating faulty computations and a second cryptography device or processor "cracking" the cryptosystem. It is also contemplated, however, that the inventive method may be performed by a single cryptography device. Party i 's
25 cryptography device generates an erroneous signature \hat{E} for the same message m . This erroneous signature may be generated, for example, while a tamper proof device 200 is placed under physical stress, such as being placed in an

extreme environment, which is likely to cause hardware faults. \hat{E} is transmitted to party j 's cryptography device or processor. Party j 's cryptography device 100 stores \hat{E} in memory (step 2).

Recall that E and \hat{E} are computed as: $E = aE_1 + bE_2 \pmod{N}$ and $\hat{E} =$
 5 $a\hat{E}_1 + b\hat{E}_2 \pmod{N}$, respectively. Because hardware faults occur with low probability, it is reasonable to assume that a hardware fault occurs during the computation of only one of \hat{E}_1, \hat{E}_2 . Thus, it is assumed that a hardware fault occurs during the computation of \hat{E}_1 , but no fault occurs during the computation of \hat{E}_2 . Thus, $\hat{E}_2 = E_2$.

10 Party j 's cryptography device now uses E and \hat{E} to obtain N in the following manner. Observe that:

$$E - \hat{E} = (aE_1 + bE_2) - (a\hat{E}_1 + b\hat{E}_2)$$

(4)

Because $\hat{E}_2 = E_2$, this equation becomes:

15 $aE_1 + bE_2 - a\hat{E}_1 - bE_2 = aE_1 - a\hat{E}_1 = a(E_1 - \hat{E}_1)$

If $E_1 - \hat{E}_1$ is not divisible by p , then:

$$\gcd(E - \hat{E}, N) = \gcd(a(E_1 - \hat{E}_1), N) = q$$

(5)

20 (step 3). Once q is obtained, party j 's cryptography device or processor 100 may easily determine N (step 4).

If the factors of N are randomly chosen by the tamper proof device 200, then it is extremely unlikely that p divides $E_1 - \hat{E}_1$. This is unlikely because $E_1 - \hat{E}_1$ can have at most $\log N$ factors. This limited number of factors is because the

lengths of E and \hat{E} limit the number of times these numbers may be divided by a smaller quantity.

By using one faulty and one correct value of the same RSA signature, the modulus used in the RSA system can be easily determined. In this attack, it makes no difference what type of fault occurs or how many faults occur in the computation of E_1 . Moreover, to determine the modulus N , only one correct and one incorrect signature of the same message needs to be received. All that is assumed is that the fault occurs in the computation modulo of one of the primes p, q only.

10 2. Cracking the RSA Signature Scheme Using Only An Incorrect Signature

In fact, one faulty signature of a known message m is sufficient to obtain N . This version of the inventive method is illustrated in Fig. 4. No correct signature of the same message is required. Let $E = m^{s_i} \bmod N$. Let \hat{E} be a faulty signature having the same fault as above, that is $E \equiv \hat{E} \bmod q$ but $E \not\equiv \hat{E} \bmod p$. Party i 's cryptography device 200 generates \hat{E} and transmits \hat{E} to party j 's cryptography device or processor 100. Party j 's cryptography device or processor receives \hat{E} . (step 1). It now follows that:

$$\gcd (M - \hat{E}^{e_i}, N) = q \quad (6)$$

where e_i is the public exponent (or public key) used to verify the decrypted signature, i.e., $E^{e_i} = m \bmod N$ (step 2). Once q is determined, party j 's cryptography device 100 or processor may easily factor N (step 3). Thus, if the cryptography device 100 or processor knows message m , it may factor the

modulus given only one faulty signature. This is important because some RSA signature implementations avoid signing the same message twice by using a "padding" technique.

If the padding is not random (i.e., the padding is an n -bit number appended to the end of the message), the message m may be determined. This improvement shows that as long as the entire signed message is known, even non-random padding protected RSA/CRT systems are vulnerable to the hardware faults attack with only a single faulty signature. If the padding is random, then the message m cannot be separated from the padding and the message m is not known. This method will not work if the message is not known.

c. Cracking the Rabin Signature Scheme

The expensive part of signing using Rabin's signature scheme is the extraction of the modular square root. This, as with the RSA signature scheme, is usually implemented using the Chinese Remainder Theorem. A cryptography device such as a smart card 200, may use its processor 202 to compute:

$$E = \sqrt{D'} \pmod{N} \quad (7)$$

20

The processor 202 first computes:

$$E_1 = \sqrt{D} \pmod{p} \text{ and } E_2 = \sqrt{D} \pmod{q} \quad (8)$$

This is done using a standard square root algorithm modulo a prime. The processor 202 may use the Chinese Remainder Theorem to compute signature E . Using the same a, b , defined above, E is determined as:

$$E = aE_1 + bE_2 \pmod{N} \quad (9)$$

5 Because the Rabin Signature Scheme may be divided into a linear equation similar to the RSA signature scheme (see eq. 4), the same attacks described above with reference to Figs. 3 and 4 may be used to crack the device using the Rabin signature scheme. Under the first attack (Fig. 3), the device signs the same message twice, one signature, E , is obtained in normal
10 conditions and is therefore the correct one. The second signature, \hat{E} , is erroneous. The erroneous signature \hat{E} may be obtained, for example, in an extreme physical environment and therefore is likely to be the erroneous signature. As described above, $\gcd(E - \hat{E}, N)$ is likely to yield a factorization of N . Under the second attack (Fig. 4), if the cryptography device 100 or other
15 processor knows document D it may factor the modulus without comparing it to a correct signature of the same message.

III. Using Register Faults To Crack Cryptosystems

20 a. Using Register Faults to Attack the Fiat-Shamir Authentication Scheme

 The Fiat-Shamir authentication scheme may be cracked by correctly guessing the value of the error caused by a register fault. Because register
25 faults have known properties (they are powers of 2 or a product of powers of

2), the error may be easily guessed by a processor. Using the Fiat-Shamir authentication scheme, a security expert's or cryptosystem designer's cryptography device 100, computer, or other processor may compare an erroneous message with a correct message to obtain a random number r selected by party i 's cryptography device, such as a tamper proof device 200 of Fig. 2. Once r is known, the security expert's or cryptosystem designer's cryptography device 100 or other processor may perform calculations to determine party i 's secret key.

Party i 's secret key, comprising a number of bits s_1, \dots, s_t , may be recovered by party j 's cryptography device by using register faults. Given t faulty runs of the protocol, party j 's cryptography device may recover the secret key bits s_1, \dots, s_t with probability of $1/2$ using $O(n^2t)$ arithmetic operations, where O is order of magnitude.

This inventive method is illustrated in Fig. 5. Assume party i uses a tamper proof device 200 on which the secret key bits are embedded in register 206 and from which the secret key bits cannot be extracted. Party j is a security expert or cryptosystem designer trying to discover the secret key bits stored in party i 's tamper proof device. To do so, party j 's cryptography device executes the protocol below several times. The protocol is performed as follows. Party i 's device generates $r^2 \bmod N$ and transmits this value to party j 's device. Party j 's cryptography device observes the value $r^2 \bmod N$ generated by party i 's device (step 1). Party j 's cryptography device then selects a subset $S \subseteq \{1, \dots, t\}$ (step 2) and observes the value $y = r \prod_{i \in S} s_i$ returned to the device.

Assume that due to a register fault in party i 's tamper proof device, one bit of a register holding a value r is inverted while party i 's device is waiting for party j to send to it subset S . In this case, party j has already received the correct $r^2 \bmod N$ value during step 1 of the protocol. However, the y value
5 computed by party i in step 3 is incorrect. Due to the register fault, party i 's device outputs:

$$\hat{y} = (r + \hat{E}) \prod_{i \in S} s_i \quad (10)$$

where \hat{E} is the value added to the register as a result of the register fault (step
10 3). This value is transmitted to party j 's device. Recall that party j 's cryptography device knows the value of $\prod_{i \in S} v_i$ from step 4 of the Fiat-Shamir protocol. Thus, party j 's cryptography device may compute:

$$(r + \hat{E})^2 = \frac{\hat{y}^2}{\prod_{i \in S} v_i} \pmod{N} \quad (11)$$

wherein $v_i = s_i^2$

15

Because \hat{E} is an inverted bit in a register, it is a binary number of low weight, i.e., a power of 2 or a sum of few powers of 2 (i.e., $\hat{E} = 2^k$ for some $1 \leq k \leq n$). Thus, party j 's cryptography device can easily determine the value of \hat{E} by selecting all possible values of \hat{E} until the correct value is determined (step 4).
20 If \hat{E} is correctly guessed, then party j 's cryptography device may recover r because:

$$(r + \hat{E})^2 - r^2 = 2\hat{E}r + \hat{E}^2 \pmod{N}$$

(12)

(step 5). This linear equation for r can be easily solved. Party j 's ability to discover the secret random r is the main observation which permits the system to be cracked.

5 Using the values of r and \hat{E} , party j 's cryptography device may compute:

$$\prod_{i \in S} s_i = \frac{\hat{y}}{r + \hat{E}} \pmod{N}$$

(13)

Thus, party j may compute the value $\prod_{i \in S} s_i$ by guessing the fault value \hat{E} and using the formula:

$$\prod_{i \in S} s_i = \frac{2\hat{E}\hat{y}}{\frac{\hat{y}^2}{\prod_{i \in S} v_i} - r^2 + \hat{E}^2} \pmod{N}$$

10

(14)

(step 6).

Party j may now verify that fault value \hat{E} was correctly guessed. Let T be the guessed value of $\prod_{i \in S} s_i$ obtained from equation (14) above. To verify that \hat{E} is correct, party j 's cryptography device checks that $T^2 = \prod_{i \in S} v_i$. There is a
 15 high probability that only one low-weight value \hat{E} exists where this relationship is satisfied. Therefore, if the relationship is satisfied, party j is very likely to have obtained the correct value of $\prod_{i \in S} s_i$.

In the unlikely event of two values \hat{E}, \hat{E}' satisfying the relation, party j may still crack the cryptosystem. Observe that the relation $(y')^2 = (r')^2 T^2$ implies that $T^2 = \prod_{i \in S} s_i$.

If there exists two low weight values \hat{E}, \hat{E}' generating two values T, T' , wherein
 5 $T \neq T'$ satisfying the relationship, then $T^2 = T'^2 \pmod{M}$. If $T \neq T' \pmod{M}$ then party j 's cryptography device already may factor N .

Assume $T = -T' \pmod{M}$. Because one of T or T' equals $\prod_{i \in S} s_i$ (i.e., one
 of \hat{E}, \hat{E}' is the correct fault value), it follows that party j now knows that $\prod_{i \in S} s_i$
 up to the sign. This is sufficient to crack the cryptosystem. Because \hat{E} is a low
 10 weight binary value, party j 's cryptography device may substitute all possible
 values for \hat{E} until the correct value is determined.

Once party j has a method for determining $\prod_{i \in S} s_i$ for various sets of S of
 party j 's cryptography device's choosing, party j may easily find party i 's secret
 key bits s_1, \dots, s_t . A simple approach is for party j 's cryptography device to
 15 construct $\prod_{i \in S} s_i$ for singleton sets, i.e., sets S containing a single element. If S
 $= \{k\}$, then $\prod_{i \in S} s_i = s_k$ and therefore s_k may be found for each k . If party i 's
 tamper proof device 200 refuses to accept a singleton set, party j may still find
 party i 's secret keys in the following manner. Party j 's cryptography device
 may select sets at random such that resulting characteristic vectors are linearly
 20 independent. A set is represented as follows: $S \subseteq (1, \dots, t)$ by its characteristic
 vector $U \in (0,1)^t$. That is, $U_i = 1$ if $i \in S$ and $U_i = 0$ otherwise. Party j picks
 sets S_1, \dots, S_t such that a corresponding set of characteristic vectors U_1, \dots, U_t
 form a $t \times t$ full rank matrix over Z_2 , where Z_2 is a ring of integers, modulo 2
 (step 7). Party j 's cryptography device then performs the Fiat-Shamir

authentication scheme above to construct the values $T_i = \prod_{i \in S} s_i$ for each of the sets S_1, \dots, S_t (step 8).

For example, party j may determine s_1 , by constructing elements $a_1, \dots, a_t \in \{0, 1\}$ such that:

$$a_1 U_1 + \dots + a_t U_t = (1, 0, 0, \dots, 0) \pmod{2}$$

5

(15)

These elements may be efficiently constructed because the vectors U_1, \dots, U_t are linearly independent over Z_2 . When all the computations are completed over the integers, the following is obtained:

$$a_1 U_1 + \dots + a_t U_t = (2b_1 + 1, 2b_2, 2b_3, \dots, 2b_t)$$

10

(16)

for some known integers b_1, \dots, b_t . Party j 's cryptography device may now compute s_1 using the formula:

$$s_1 = \frac{T_1^{a_1} \dots T_t^{a_t}}{v_1^{b_1} \dots v_t^{b_t}} \pmod{N}$$

(17)

(step 9). Recall that the values $v_i = s_i^2 \pmod{N}$ are publicly available. The values s_2, \dots, s_k may be obtained using the same procedure.

15

The procedure above made use of t faults and took $O(n^2 t)$ arithmetic operations. The faults occur while party j 's device is waiting for a challenge from the outside world. Consequently, the security expert knows when the register faults are induced.

The Fiat-Shamir scheme may be modified. Recall that in the Fiat-Shamir protocol set out above, in step 2, party i sends $r^2 \pmod{M}$ to party j ; and in step 4, party j verifies party i 's identity by checking that $y^2 = r^2 \prod_{i \in S} v_i \pmod{M}$. If this protocol is modified to use higher powers instead of just squaring the values, the inventive method illustrated in Fig. 5 may still be used to crack the modified scheme.

Assume that the modified scheme uses a publicly known exponent e instead of squaring. As before, party i 's secret key is a set of invertible elements (bits) $s_1, \dots, s_t \pmod{M}$. Party i 's public key is a set of bits $v_1 = s_1^e, \dots, v_t = s_t^e \pmod{M}$. Party i authenticates itself to party j using the following protocol:

1. Party i 's cryptography device, computer, or other processor selects a random r , generates $r^e \pmod{M}$ and transmits this value to party j via an I/O.
2. Party j 's cryptography device or other processor selects a random subset $S \subseteq \{1, \dots, t\}$, and sends the subset to party i 's device via an I/O.
3. Party i 's cryptography device computes $r \prod_{i \in S} s_i$.
4. Party j 's cryptography device verifies party i 's identity by checking that $y^e = r^e \prod_{i \in S} v_i \pmod{M}$.

When $e = 2$, this protocol is the same as the original Fiat-Shamir protocol described above. Using the methods described above, party j may obtain the value $L_1 = r^e \pmod{M}$ and $L_2 = (r + \hat{E})^e \pmod{M}$. As above, assume that party j 's cryptography device has correctly guessed the value of \hat{E} . Given these two

values, party j 's device may recover r by observing that r is a common root of the two polynomials: $x^e = L_1 \pmod{M}$ and $(x + E)^e = L_2 \pmod{M}$. Furthermore, r is likely to be the only common root of the two polynomials. Consequently, when the exponent e is small, e.g., $e < n^2$, party j may recover

5 r by computing the greatest common divisor of the two polynomials. Once party j has a method for computing r he can recover the secret key bits s_1, \dots, s_t as discussed above.

b. Using Register Faults to Crack Schnorr's Authentication Scheme

Using register faults, secret integer s_i of the Schnorr authentication

10 scheme may be extracted from party i 's cryptography device. When p is an n -bit prime number, the attack requires $n \log n$ faulted values and $O(n^2)$ arithmetic operations. This is done as follows:

Let p be an n -bit prime number. Given $n \log n$ faulty runs of the protocol, secret key s_i may be recovered with probability at least $\frac{1}{2}$ using $O(n^3)$ arithmetic

15 operations.

Party j wishes to extract the secret information stored in the device. Party j 's cryptography device or other processor selects a random challenge t . The same challenge will be used in all invocations of the protocol. Because party i 's device cannot possibly store all challenges given to it thus far, it

20 cannot possibly know that party j is always providing the same challenge t . The attack will enable party j to determine the value $t \cdot s_i \pmod{p}$ from which the secret value s_i can be easily found. For simplicity, set $x = t s_i \pmod{p}$ and assume that $g^x \pmod{p}$ is known to party j 's device.

Suppose that due to a register fault in party j 's device, one of the bits of the register 206 holding the value r is flipped while the device is waiting for party j 's device to send it the challenge t . More precisely, when the third phase of the protocol is executed, the party j 's device finds $\hat{r} = r + 2^i$ in the register
 5 holding r . Consequently, party j 's device will output $\hat{d} = \hat{r} + x \bmod p$. Party j 's device may then determine the value of i (the fault position) by trying all possible values $i = 0, \dots, n$ until finding an i satisfying:

$$g^{\hat{d}} = g^{r+2^i} g^x \pmod{p}$$

(18)

10 Assuming a single bit flip, there is exactly one such i . The above identity proves to party j that $\hat{r} = r + 2^i$ showing that the i th bit of r is 1.

This information permits party j to recover x in time $O(n^2)$. Assuming that the register faults occur at uniformly and independently chosen locations in the register r , s_i may be recovered in time $O(n^2)$. It follows that with
 15 probability at least $\frac{3}{4}$ that a fault will occur in every bit position of the register r . In other words, for every $1 \leq i \leq n$ there exists an $r^{(i)}, \dots, r^{(k)}$ such that the i th bit of $r^{(i)}$ is known to party j (the first bit is the LSB).

To recover s_i , party j 's cryptography device first guesses the $\log 8n$ bit strings until the correct one is found. Let X be the integer that matches x on
 20 the most significant $\log 8n$ bits and is zero on all other bits. Party j 's device correctly guesses the value of X . Party j 's device may recover the rest of s_i

starting with the LSB. Inductively, suppose party j 's device has already determined bits s_{i-1}, \dots, s_2, s_1 of x . (Initially $i = 1$). Let:

$$Y = \sum_{j=1}^{i-1} 2^j x_j \quad (19)$$

- 5 To determine bit s_n , party j 's device uses $r^{(i)}$ of which it knows the i th bit and the value of $x + r^{(i)}$. Let b be the i th bit of $r^{(i)}$. Then

$$x_i = b \oplus i\text{'th bit}(x + r^{(i)} - Y - X \bmod p-1) \quad (20)$$

- assuming no wrap around, i.e., $0 \leq x + r^{(i)} - Y - X < p-1$ (the remainder cannot be greater than the number being divided). Because $x - X < p/8n$, wrap around may
10 occur only if $r^{(i)} > (1-1/8n)p$. Since the r s are independently and uniformly chosen in the range $[0, p)$, the probability that this does not happen in all n iterations of the method is more than $3/4$.

- Once X is correctly determined, the method runs in linear time and outputs x with probability at least $1/2$. (The reason for the $1/2$ is that all bits of
15 r should be "covered" by faults and all r_i should not be too large. Both events are satisfied with probability at least $1/2$.) Of course, once a candidate x is found, it can be easily verified using the public data. There are $O(n)$ possible values for X and thus, the running time of this step is $O(n^2)$.

- This attack also works in the case of multiple bit flips of the register r .
20 As long as the number of bit flips is constant, their exact location can be found and used by party j . Note that the faults occur while party i 's device is waiting

for a challenge from the outside world. Consequently, party j knows exactly at what time the faults should be induced.

c. Using Register Faults to Crack RSA Implementations

Register faults may be used to break other RSA implementations that are not based on the Chinese Remainder Theorem. Let N be an n -bit RSA composite and s_i be a secret exponent. The exponentiation function $x \rightarrow x^{s_i} \bmod N$ may be computed using either of the following conventional methods 600, 650 illustrated in Figs. 6A and 6B:

- Method I (Fig. 6A)

10 init $y \leftarrow x$; $z \leftarrow 1$ (step 602).
 main For $k = 1, \dots, n$ (steps 604, 610).
 if k th bit of s is 1 (step 606) then $z \leftarrow zy \bmod M$ (step 608).
 $y \leftarrow y^2 \bmod M$ (step 610).

15 Output z (step 612).

- Method II (Fig. 6B)

 init $z \leftarrow x$ (step 652).
 main For $k = n-1$ down to 1 (steps 654, 662).
 if k th bit of s is 1 (step 656) then $z \leftarrow z^2x \bmod M$ (step 658)
 otherwise, $z \leftarrow z^2 \bmod M$ (step 660).
 Output z (step 664).

For both methods 600, 650, given several faulty values by a cryptography device, a security expert's or cryptosystem designer's cryptography device, computer, or processor may obtain the secret exponent in polynomial time. In this version of the invention, faulty values are obtained
 5 in the presence of register faults. This attack uses erroneous signatures of randomly chosen messages; the attacker need not obtain the correct signature of any of the messages. Furthermore, an attacker's cryptography device or processor need not obtain multiple signatures of the same message.

With probability of at least $1/2$, the secret exponent s_i can be extracted
 10 from a device (such as smart card 200) implementing the first exponentiation algorithm by collecting $(n/m) \log n$ faults and $O(2^m \cdot n^3)$ RSA encryptions, for any $1 \leq m < n$. For a small public exponent e_i this takes $O(2^m \cdot n^4)$ time. For a random e_i it takes $O(2^m \cdot n^5)$ time.

The following faults are used: let m be a message to be signed and m
 15 $\in Z_N$, where Z_N is a ring of integers modulo N . Suppose that a register fault occurs at a single random point during the computation of $m^{s_i} \bmod N$. That is, at a random point in the computation one of the bits stored in a register (such as register 206) flips. The resulting erroneous signature is \hat{E} . An ensemble of such erroneous signatures enables one to recover the secret exponent s_i . Even
 20 if other types of faulty signatures are added to the ensemble, they do not confuse the inventive method.

Let $l = (n/m) \log n$ and let $m_1, \dots, m_l \in Z_N$ be a set of random messages. Set $E_i = m_i^{s_i} \bmod N$ to be the correct signature of a message m_i . Let \hat{E}_i be an erroneous signature of the message m_i . A register fault occurs at exactly one

point during the computation of \hat{E}_i . Let k_i be the value of k (recall k in the counter in Method 1, 600 above) at the point at which the fault occurs. Thus, for each faulty signature \hat{E}_i there is a corresponding k_i indicating the time at which the fault occurs. The messages may be sorted by a processor (i.e., 202 of Fig. 2) so the $1 \leq k_1 \leq k_2 \leq \dots \leq k_l < n$. The time at which the faults occur is chosen uniformly (among the n iterations) and independently at random. It follows that given l such faults, with the probability at least half $k_{i+1} - k_i < \epsilon$ for all $i = 1, \dots, l$. Since the location of the faults are unknown, the values k_i are also unknown.

Let $s_i = s_n, s_{n-1}, \dots, s_1$ be the bits of the secret exponent s_i where s_n is the MSB and s_1 is the LSB. Each of the bits in s_i is recovered one-by-one. A block of these bits at a time may be recovered starting with the MSBs. Assume bits $s_n, s_{n-1}, \dots, s_{k_i}$ for some i are known. Initially $i = l + 1$ indicating that no bits are known and it is desired to determine the next bit. Bits $s_{k_{i-1}}, s_{k_{i-2}}, \dots, s_{k_{i-1}}$ may be recovered in the following manner. All possible bit vectors are tried by a cryptography device or computer until the correct one is found. Note that the location within s_i of each bit s_n, s_{n-1}, \dots is unknown. Because even the length of the block is unknown, all possible lengths are tried. The inventive method for determining the next bit may be performed by a cryptography device or computer processor using the method 700 illustrated in Fig. 7 and set out below.

1. Initialize $r = 0$ (step 702);
2. For all lengths $r = 0, 1, 2, 3, \dots$ (step 704) do:
3. For all candidate r -bit vectors $U_{k_{i-1}}, U_{k_{i-2}}, \dots, U_{k_{i-r}}$ (step 706) do:

4. Set:

$$w = \sum_{j=k_i}^n s_j 2^j + \sum_{j=k_i-r}^{k_i-1} u_j 2^j. \quad (21)$$

In other words, w matches the bits of s and U at all known bit positions and is zero everywhere else (step 708).

5. Test if the current candidate bit vector is correct by checking if one of the erroneous signatures $\hat{E}_j, j = 1, \dots, l$ satisfies (step 710):

$$\exists e \in \{0, \dots, n\} \text{ s.t. } (\hat{E}_j \pm 2^e m_j^w)^{e_i} = m_j \pmod{N} \quad (22)$$

wherein e is the public exponent. (The \pm means that the condition is satisfied if it holds with either a plus or a minus.) This step means that all combinations of bits using this erroneous bit are tested until the proper location is found. The inverse of the erroneous bit is the correct bit in that location.

6. If a signature satisfying the above condition is found (step 712), the cryptography device outputs $u_{k_{i-1}}, u_{k_{i-2}}, \dots, u_{k_{i-1}}$ and stops (step 714) for that r . At this point it is determined that $k_{i-1} = k_r r$ and $s_{k_{i-1}}, s_{k_{i-2}}, \dots, s_{k_{i-1}} = u_{k_{i-1}}, u_{k_{i-2}}, \dots, u_{k_{i-1}}$. If a signature satisfying the condition is not found, another candidate vector is tried (step 716).

Steps 706-716 are repeated for each r between 0 and n (steps 718, 704). The condition at step 710 is satisfied by the correct candidate $u_{k_{i-1}}, u_{k_{i-2}}, \dots, u_{k_{i-1}}$. Recall that \hat{E}_{i-1} is obtained from a fault at the k_{i-1} st iteration. At the k_{i-1} st iteration, the value of z was changed to $\hat{z} \leftarrow z \pm 2^e$ for some public

exponent e . Notice that at this point $\hat{E}_{i-1} = zM^{w_{i-1}}$. From that point on no fault occurred and therefore the signature \hat{E}_{i-1} satisfies:

$$\hat{E}_{i-1} = \hat{z}Mw_{i-1} = \hat{E}_{i-1} \pm 2^e Mw_{i-1} \pmod{N} \quad (23)$$

When in step 710 the signature \hat{E}_{i-1} is correct, it properly verifies when raised to the public exponent e . Consequently, when the correct candidate is tested, the faulty signature \hat{E}_{i-1} guarantees that it is accepted. There is a high probability that a wrong candidate will not pass the test.

Note that not all of the bits need to be determined in this manner. For example, if 450 bits of a 512 bit key are determined through register faults, the unknown 62 bits may be tested by trying different combinations of these unknown bits. Each combination is used as the secret key s_i and a received message is attempted to be decrypted. If the message is properly decrypted, the combination used is correct.

If the method obtains both the faulty and correct signature of each message m_i , the running time is improved to $O(2^m n^2)$ arithmetic operations modulo N which takes time $\tilde{O}(2^m n^3)$. This follows since the error location \hat{E} can be easily found using a lookup table of powers of 2 mod N .

Using these algorithms, given $n \log n$ faulted values a cryptography device, computer, or other processor may recover the secret exponent in polynomial time. That is, for a message $m \in Z_N$, given $n \log n$ faulted values output by a device computing the function $m^{s_i} \pmod{N}$, the secret exponent s_i may be recovered in polynomial time in n . A cryptography device or computer

may employ either one of the exponentiation methods above. Note that faulted values for this version of the present invention are register faults.

The method illustrated in Fig. 7 may also be used to crack ElGamal's public key cryptosystem. In ElGamal's public key cryptosystem, party i selects a secret exponent s_i and publishes the public key $y = g^{s_i} \bmod p$, where p is an n -bit prime number. To send a message m to party i , party j 's device selects a random number b and sends to party i two values $E_1 = y^b m \bmod p$ and $E_2 = g^b \bmod p$. Party i decrypts the message by computing $E_1/E_2^{s_i} \bmod p$; that is:

$$m = \frac{g^{sb} m \pmod{p}^2}{g^{sb} \bmod p \bmod p}$$

(24)

Assume party i uses a smart card 200 for decryption. The smart card contains in a register 206 the secret exponent s_i and will output the plain text message. The attack on the RSA implementation discussed above permits the extraction of the secret exponent s_i from the smart card.

IV. Providing Cryptosystems and Cryptography Devices Which Resist Tampering Due to Hardware Faults

In addition to the attacks described above, it is believed that hardware fault based attacks may be performed on the following cryptosystems:

1. DES;
2. IDEA;

3. RC5;
4. FEAL; and
5. Skipjack.

These are secret (or symmetric) key systems, not public key systems.

5 Nevertheless, hardware based faults may permit these systems to be cracked.

Regardless of whether a cryptosystem is a public or private key system, vulnerability to a fault-based attack results in a cryptosystem which has questionable security, regardless of whether the fault is generated by machine imperfection, software bugs, intentionally induced faults, or faults planted at
10 the chip level during design and/or manufacturing.

The Secure Electronic Transmission (SET) standard for on-line Internet bank card transactions address faulty computations. The standard provides that if a received message fails an authentication test, an error message is returned. Thus, erroneous security data is not expected, but acknowledged.
15 This acknowledgement is available to eavesdroppers. Ideally, an assurance of zero faulty computation is the best protection. However, as discussed above, it is well-accepted that no computing system is entirely error-free. As a result, verifying cryptographic computations before outputting results is preferred.

A simple way to avoid cracking due to hardware faults is to check the
20 output of a computation before releasing it. This may require recomputing functions, which may sometimes result in an expensive or time consuming process which may be unacceptable.

Authentication schemes may be attacked based on register faults in the internal memory of a cryptography device. Protection against such an attack

may include (1) detecting the fault; and (2) correcting the fault. Because a register fault changes the stored data, the device may have computed the (temporarily incorrect) data correctly. This recomputing the data may not reveal an error. In multi-round authentication schemes such as Fiat-Shamir, for example, error detection/correction bits, such as CRC bits, may be added to protect the validity of the stored data.

Signature schemes may be attacked based on a fault during the computation of the signature. One way to overcome this attack is to verify the signature before outputting it. The device generating the signature may, for example, apply the signature verification algorithm on the signature before transmission. For example, this may be done in RSA signatures by applying the public key to the signature. A second way to overcome this attack is to pad the signed message with random bits. This may prevent an adversary from obtaining two copies of an identical message.

An alternative approach to protecting RSA computations which do not use the Chinese Remainder Theorem is the use of blinding. To compute $x^s \bmod N$, the cryptography device first picks a random number r and computes $y = r^e \bmod N$, where e is the public exponent: The device computes $(xy)^s / r \bmod N$. The result is $x^s \bmod N$.

These attacks may be used in the design and testing of existing and future cryptosystems and cryptography devices. Although future cryptography devices may have the same overall structure as seen in Figs. 1A and 2, such devices should preferably be modified to be impervious to hardware-fault based attacks. One way to design such a device may be to implement one of the

software solutions described above. Another way to design such a device is protect the device's internal storage from extreme environments by providing shielding or other hardware solutions.

A cryptography device may be verified to be impervious to a hardware fault-based attack by subjecting the device to one or more of the attacks described above. The cryptography device may be verified to be secure against these attacks by verifying that an adversary cannot determine secret information stored in the cryptography device.

10 V. Conclusion

Methods for cracking public key cryptosystems are described wherein an erroneous calculation is used to infer secret information. The inventive method is a creative use of a cryptosystem device's miscalculations. Because it is believed that all computers are prone to error, even cryptosystem servers stored in a secure environment are not protected from the inventive method of testing public key cryptosystems.

The inventive methods are particularly useful to security experts and cryptosystem designers. Existing and proposed cryptosystems are cryptography devices should be impervious to the attacks described. If not, the system or device should be modified or discarded. Smart cards, for example, should now not only conceal their inner circuitry (to avoid revealing its secret key), but also be fault resistant and/or check computed values before transmission, to avoid revealing secret information.

CLAIMS

We claim:

- ✓ 1. A method for testing security of a cryptography device performing a cryptographic algorithm, comprising the steps of:
- 5 a. generating a faulty computation in the cryptography device;
- b. receiving the faulty computation in a processor; and
- c. using the faulty computation, the processor determining heretofore
secret information stored in the cryptography device.
- 10 2. The method of claim 1, wherein the faulty computation is intentionally generated.
3. The method of claim 2, wherein the faulty computation is intentionally generated by subjecting the cryptography device to a physical stress.
- 15 4. The method of claim 3, wherein the step of subjecting the cryptography device to physical stress further includes subjecting the cryptography device to at least one of radiation, an atypical voltage level, and a higher clock speed than the cryptography device was designed to accommodate.
- 20 5. The method of claim 1, further comprising the step of transmitting the faulty computation from the cryptography device to a second cryptography device housing the processor.

6. The method of claim 1, wherein the cryptographic algorithm generates a digital signature which may be separated into linear components, wherein the step of determining heretofore secret information further comprises the processor comparing an erroneous signature having the generated fault on a digital message with a correct digital signature on the same digital message.

7. The method of claim 1, wherein the faulty computation is generated by inverting at least one bit stored in a register of the cryptography device.

8. The method of claim 7, wherein the step of determining heretofore secret information further comprises the processor comparing a correct value and an erroneous value containing the induced fault to determine the secret information.

9. The method of claim 1, wherein the cryptographic algorithm generates a digital signature, wherein the method further comprises the steps of:

a. the step of generating a faulty computation further comprises inducing a faulty computation in a plurality of digital messages; and

b. the step of determining heretofore secret information further comprises:

(i) the processor using a first fault to determine heretofore secret information;

(ii) the processor constructing sets of data for the cryptography device;

(iii) the processor receiving from the cryptography device responses to the sets of data; and

5 (iv) the processor using the heretofore secret information and the responses to determine a secret key.

10. The method of claim 1, wherein the cryptographic algorithm is an authentication algorithm, wherein:

10 a. the step of receiving the faulty computation comprises receiving
the faulty computation in response to a challenge; and

b. the step of determining heretofore secret information further comprises the processor using the faulty computation to determine a single bit of heretofore secret information; and

15 c. repeating steps (a) and (b) above to determine a plurality of bits of
secret information.

11. A method for testing the security of a cryptography device which performs a cryptographic algorithm which generates a digital signature which may be separated into linear components, the method comprising the steps of:

- a. storing in a memory a correct digital signature E for a message m generated by the cryptography device;

b. storing in the memory an incorrect digital signature \hat{E} for the message m generated by the cryptography device; and

- c. using E and \hat{E} , a processor determining heretofore secret information.

12. The method of claim 11, wherein method is performed by the processor,

5 the method further comprising the steps of:

- a. the cryptography device sending E to the processor; and
- b. the step of determining heretofore secret information further comprises the first processor determining heretofore secret information stored in the cryptography device.

10

13. The method of claim 11, wherein the step of determining heretofore secret information further comprises the processor determining secret information q stored in the cryptography device using:

15
$$\gcd(E - \hat{E}, N) = q$$

wherein N is a product of prime numbers, and one of the prime numbers is q .

14. A method for testing the security of a cryptography device performing cryptographic authentication algorithm, the method comprising the steps of:

20

- a. receiving from the cryptography device a value $r^2 \bmod N$, wherein r is a random number and N is a secret value which is a product of prime numbers;

- 5
- b. generating a subset of integers S and providing the subset S to the cryptography device;
 - c. receiving from the cryptography device $\hat{y} = (r + \hat{E}) \prod_{i \in S} s_i$ in response to the subset S , wherein \hat{y} is an erroneous value, s_i is a secret exponent used to encrypt, and \hat{E} is a value added to r due to an error;
 - d. a processor determining a value of \hat{E} by computing:

$$(r + \hat{E})^2 = \frac{\hat{y}^2}{\prod_{i \in S} v_i} \pmod{N}$$

10 wherein $v_i = s_i^2$

- e. the processor determining a value of r by computing:

$$(r + \hat{E})^2 - r^2 = 2\hat{E}r + \hat{E}^2 \pmod{N}$$

and

- 15
- f. using the values of \hat{E} and r , the processor determining s_i by computing:

$$\prod_{i \in S} s_i = \frac{\hat{y}}{r + \hat{E}} \pmod{N}.$$

15. The method of claim 14, wherein the step of determining s_i further includes the step of the cryptography device computing:

$$\prod_{i \in S} s_i = \frac{2\hat{E}\hat{Y}}{\frac{\hat{Y}^2}{\prod_{i \in S} v_i} - T^2 + \hat{E}^2} \pmod{N}.$$

5 16. The method of claim 14, further comprising the step of verifying whether the value of \hat{E} is correct.

17. The method of claim 16, wherein the step of verifying further includes the step of using the subset S to determine whether the value of \hat{E} satisfies the
10 relation $(y')^2 = (r')^2 T^2$;
wherein T is a guessed value for $\prod_{i \in S} s_i$

18. The method of claim 14, further comprising the steps of:

- a. the processor generating a plurality of subsets S ;
- 15 b. the processor receiving a value in response to each subset S ; and
- c. using known values and the response value to each subset S , the processor determining heretofore secret information.

19. The method of claim 18, wherein the step of generating the plurality of
20 subsets S further comprises generating singleton sets.

20. A method for testing the security of a cryptography device performing a cryptographic authentication algorithm by determining secret information comprising a number of bits, the method comprising the steps of:

- a. a processor obtaining an erroneous digital signature \hat{E} ;
- 5 b. the processor selecting a block length;
- c. the processor determining a candidate vector w that matches all known bits of the secret information and is zero everywhere else;
- d. the processor determining if the candidate vector w is correct;
- e. if the candidate vector w is correct, the processor outputting a value for the selected block length; and
- 10 f. if the candidate vector w is incorrect, the processor determining another candidate vector.

21. The method of claim 20, wherein steps (c) - (f) are performed for a plurality of block lengths.

22. The method of claim 20, wherein the step of determining the candidate vector w further comprises determining:

$$w = \sum_{j=k_i}^n s_j 2^j + \sum_{j=k_i-r}^{k_i-1} u_j 2^j.$$

20 wherein k_i is a time at which an error may have occurred; s_j is a bit which may be incorrect; r is a possible blocklength; and u is a bit which may be incorrect.

23. The method of claim 20, wherein the step of determining if the candidate vector w is correct further comprises determining:

$$\exists e \in \{0, \dots, n\} \text{ s.t. } (\hat{E}_j \pm 2^e m_j^w)^{e_i} = m_j \pmod{N}$$

wherein e = a public exponent;

5 n = a number of bits in the secret information;

m_j = is a message;

e_i = is a public signature verification exponent; and

N = a product of prime numbers.

10 24. A method for testing the security of a first cryptography device performing cryptographic authentication algorithm by using a second cryptography device to determine secret information comprising a number of bits stored in the first cryptography device, the method comprising the steps of:

15 a. the second cryptography device sending to the first cryptography device a challenge t ;

b. the first cryptography device receiving t and generating a response $u = r + ts \pmod{p}$, wherein:

r is a random number selected by the first cryptography device;

20 s is the first cryptography device's secret key; and

p is a large prime number;

c. the second cryptography device receiving u ;

d. the first cryptography device receiving t again and generating a response $\hat{u} = \hat{r} + x \pmod{p}$, wherein:

\hat{r} is an erroneous value of r and x is $ts \bmod p$; and

- e. the second cryptography device receiving \hat{d} and determining a location of the error.

5 25. The method of claim 24, wherein the step of determining the location of the error further comprises the steps of trying all possible locations of the error.

26. The method of claim 25, wherein the step of trying all possible locations further includes the step of determining which location for the error satisfies:

10
$$g^{\hat{d}} = g^{2i} g^r g^x \pmod{p}$$

wherein:

g is a generator of Z_p^* ; and

i is a location of the error.

- ✓ 15 27. A cryptography device produced according to the steps of:
- a. generating a faulty computation in the cryptography device;
 - b. receiving the faulty computation in a processor; and
 - c. using the faulty computation, verifying that the processor cannot determine secret information stored in the cryptography device.

20

28. The device of claim 27, further comprising providing the cryptography device before generating the faulty computation.

29. The device of claim 27, wherein the faulty computation is intentionally generated during testing of the cryptography device.

30. The device of claim 29, wherein the faulty computation is intentionally generated during testing of the cryptography device by subjecting the cryptography device to a physical stress.

31. The device of claim 29, wherein the step of subjecting the cryptography device to physical stress further includes subjecting the cryptography device to at least one of radiation, an atypical voltage level, and a higher clock speed than the cryptography device was designed to operate on.

32. The device of claim 27, wherein the cryptography device performs a cryptographic algorithm which generates a digital signature which may be separated into linear components, wherein the step of verifying that the processor cannot determine secret information further comprises the processor verifying that an erroneous digital signature having the generated fault on a digital message cannot be compared with a correct signature on the same digital message.

33. The device of claim 27, wherein the faulty computation is generated by inverting at least one bit stored in a register of the cryptography device.

34. The device of claim 33, wherein the step of verifying that the processor cannot determine heretofore secret information further comprises verifying that the processor cannot compare a correct value and an erroneous value containing the induced fault to determine the secret information.

5

35. A cryptography device that is impervious to a hardware fault-based attack, which attack comprises the steps of:

- a. generating a faulty computation in the cryptography device;
- b. receiving the faulty computation in a processor; and
- 10 c. using the faulty computation, the processor determining secret information stored in the cryptography device.

36. The device of claim 35, further comprising providing the cryptography device before generating the faulty computation.

15

37. The device of claim 35, wherein the faulty computation is intentionally generated during testing of the cryptography device.

38. The device of claim 37, wherein the faulty computation is intentionally
20 generated during testing of the cryptography device by subjecting the cryptography device to a physical stress.

39. The device of claim 38, wherein the step of subjecting the cryptography device to physical stress further includes subjecting the cryptographic device to at least one of radiation, an atypical voltage level, and a higher clock speed than the cryptography device was designed to operate on.

5

ABSTRACT

A useful method of verifying the integrity of a cryptosystem involves using erroneous outputs to obtain secret information. In certain signature schemes which use the Chinese Remainder Theorem, a correct signature of a message and an erroneous signature of the same message permit the modulus to be easily obtained. If the content of the message is known, such cryptosystems may be cracked with only an erroneous signature of the message. Certain other authorization schemes may be cracked by analyzing a number of erroneous outputs caused by a particular type of error called a "register fault." A security expert or cryptosystem designer may intentionally induce a tamper proof device generate a faulty computation by subjecting the device, such as a smart card, to physical stress, such as certain types of radiation, atypical voltage levels, or a higher clock rate than the device was designed to accommodate. Cryptosystems should be impervious to the attacks described herein. If not, the system should be modified or discarded.

Fig. 1A (Prior Art)

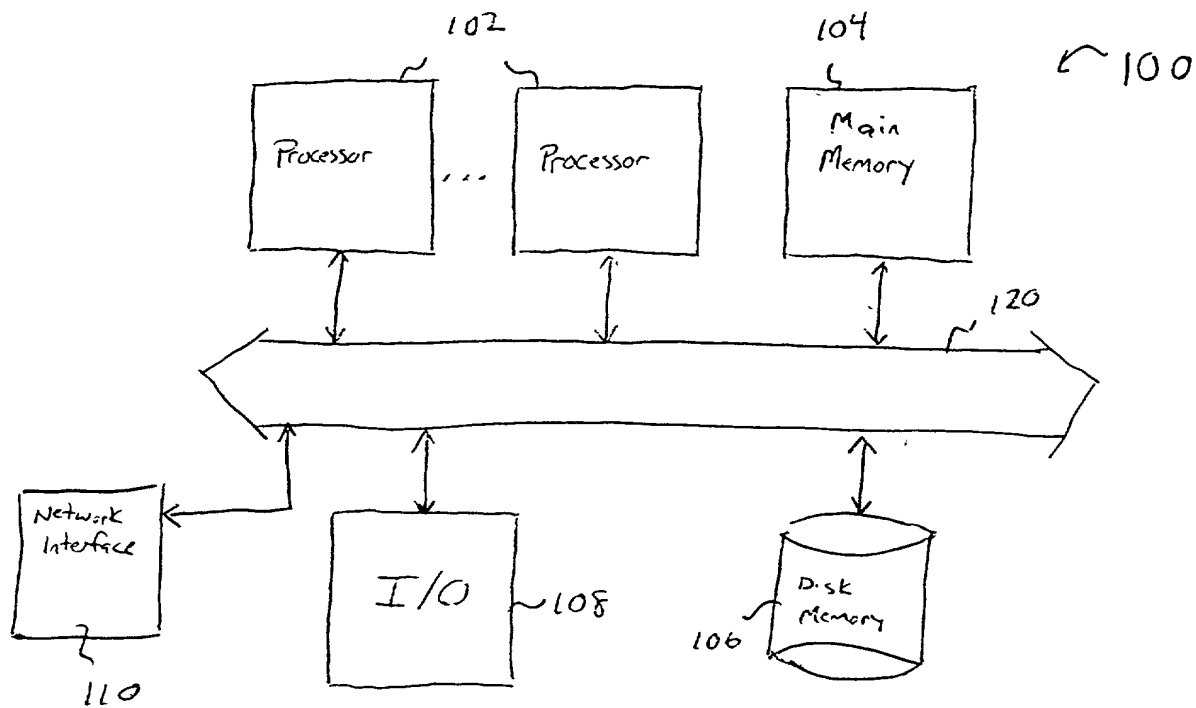
 $\frac{1}{8}$ 

Fig. 2 (Prior Art)

↪ 200

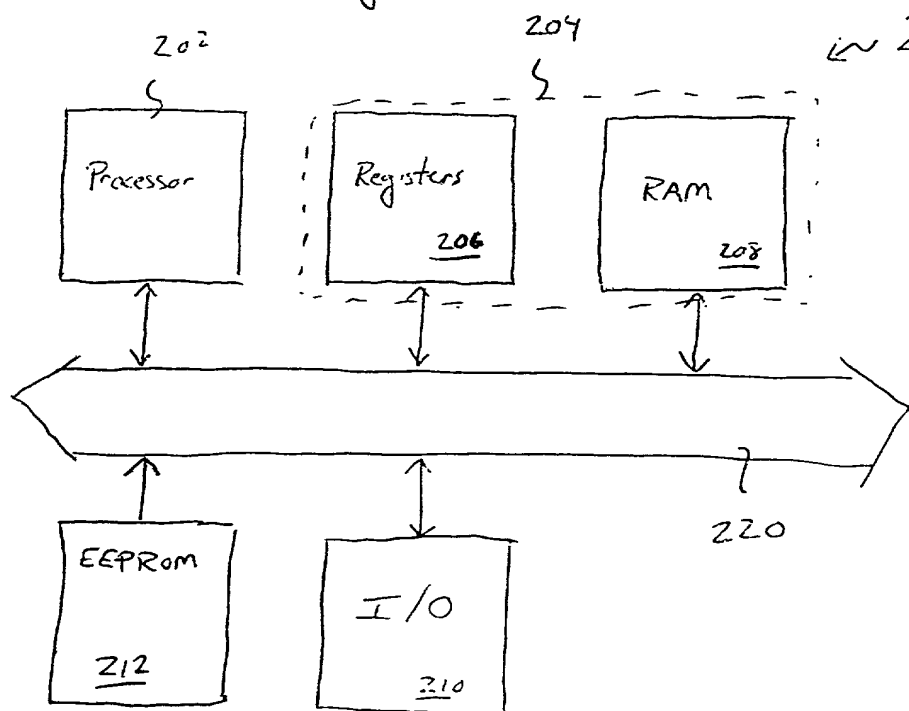
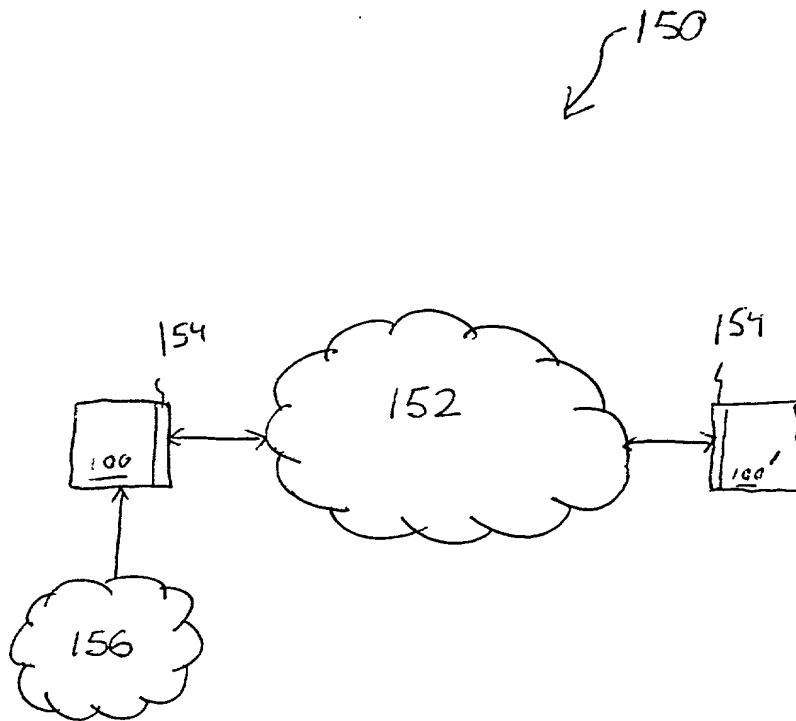


Fig. 1B (Prior Art)



Party *i*

Step

- Party i 's cryptography device generates a correct signature E for message m and transmits E to party j 's cryptography device.

Party i 's cryptography device generates incorrect signature \hat{E} for the same message m and transmits \hat{E} to party j 's cryptography device. _____

Party j's cryptography device determines

$$a(E_1 - \hat{E}_1);$$
$$\gcd(E - \hat{E}, N) = q.$$

(4)

Fig. 4

Step

Party *i*

Party *j*

(1)

Party *i*'s cryptography device generates erroneous signature \hat{E} for known message *m* (i.e., message *m* is generated without padding or with non-random padding) and transmits \hat{E} to party *j*'s cryptography device.

(2)

Party *j*'s cryptography device receives \hat{E} .

↓

Party *j*'s cryptography device determines $\gcd(M - \hat{E}^{e_i}, N) = q$.

↓

(3)

Having determined *q*, party *j*'s cryptography device determines *N*.

09546940 09546940

Fig. 5

Step

Party *i*

Party j

(1)

Party i 's cryptography device selects a random r , generates $r^2 \bmod N$, and transmits this value to party j 's cryptography device.

Party j 's cryptography device observes the value $r^2 \bmod N$.

(2)

Party j 's cryptography device generates a random subset $S \subseteq \{1, \dots, t\}$ and transmits S to party i 's cryptography device.

(3)

While waiting to receive S from party j 's device, a value in party i 's device is inverted. After receiving S from party j 's device, party i 's device generates $\hat{y} = (r + \hat{E}) \prod_{i \in S} s_i$. Party i 's device transmits \hat{y} to party j 's device.

Party j 's device receives \hat{y} .

(4)

Party j 's device determines \hat{E} by finding an \hat{E} satisfying

$$\frac{(r+E)^2}{\prod_{i \in S} v_i} \pmod{N}$$

(5)

This is possible because $E = 2^k$ for some $1 \leq k \leq n$

Party j's
device may determine r
using:

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100	2101	2102	2103	2104	2105	2106	2107	2108	2109	2110	2111	2112	2113	2114	2115	2116	2117	2118	2119	2120	2121	2122	2123	2124	2125	2126	2127	2128	2129	2130	2131	2132	2133	2134	2135	2136	2137	2138	2139	2140	2141	2142	2143	2144	2145	2146	2147	2148	2149	2150	2151	2152	2153	2154	2155	2156	2157	2158	2159	2160	2161	2162	2163	2164	2165	2166	2167	2168	2169	2170	2171	2172	2173	2174	2175	2176	2177	2178	2179	2180	2181	2182	2183	2184	2185	2186	2187	2188	2189	2190	2191	2192	2193	2194	2195	2196	2197	2198	2199	2200	2201	2202	2203	2204	2205	2206	2207	2208	2209	2210	2211	2212	2213	2214	2215	2216	2217	2218	2219	2220	2221	2222	2223	2224	2225	2226	2227	2228	2229	2230	2231	2232	2233	2234	2235	2236	2237	2238	2239	2240	2241	2242	2243	2244	2245	2246	2247	2248	2249	2250	2251	2252	2253	2254	2255	2256	2257	2258	2259	2260	2261	2262	2263	2264	2265	2266	2267	2268	2269	2270	2271	2272	2273	2274	2275	2276	2277	2278	2279	2280	2281	2282	2283	2284	2285	2286	2287	2288	2289	2290	2291	2292	2293	2294	2295	2296	2297	2298	2299	2300	2301	2302	2303	2304	2305	2306	2307	2308	2309	2310	2311	2312	2313	2314	2315	2316	2317	2318	2319	2320	2321	2322	2323	2324	2325	2326	2327	2328	2329	2330	2331	2332	2333	2334	2335	2336	2337	2338	2339	2340	2341	2342	2343	2344	2345	2346	2347	2348	2349	2350	2351	2352	2353	2354	2355	2356	2357	2358	2359	2360	2361	2362	2363	2364	2365	2366	2367	2368	2369	2370	2371	2372	2373	2374	2375	2376	2377	2378	2379	2380	2381	2382	2383	2384	2385	2386	2387	2388	2389	2390	2391	2392	2393	2394	2395	2396	2397	2398	2399	2400	2401	2402	2403	2404	2405	2406	2407	2408	2409	2410	2411	2412	2413	2414	2415	2416	2417	2418	2419	2420	2421	2422	2423	2424	2425	2426	2427	2428	2429	2430	2431	2432	2433	2434	2435	2436	2437	2438	2439	2440	2441	2442	2
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$$6/8$$

(7)

7
(8)

8
(9)

Using the sets constructed above, party j 's device performs the Fiat-Shamir authentication scheme, providing the sets to party i 's device.

(9)

?

Fig. 6A (Prior Art)

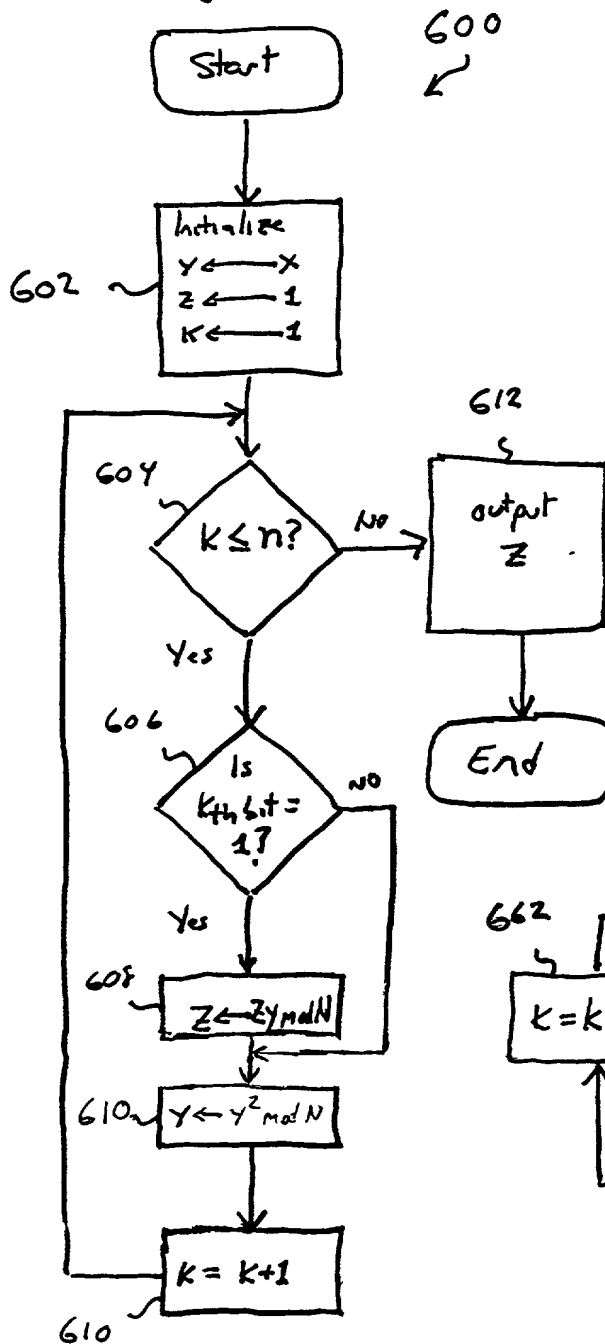


Fig. 6B (Prior Art)

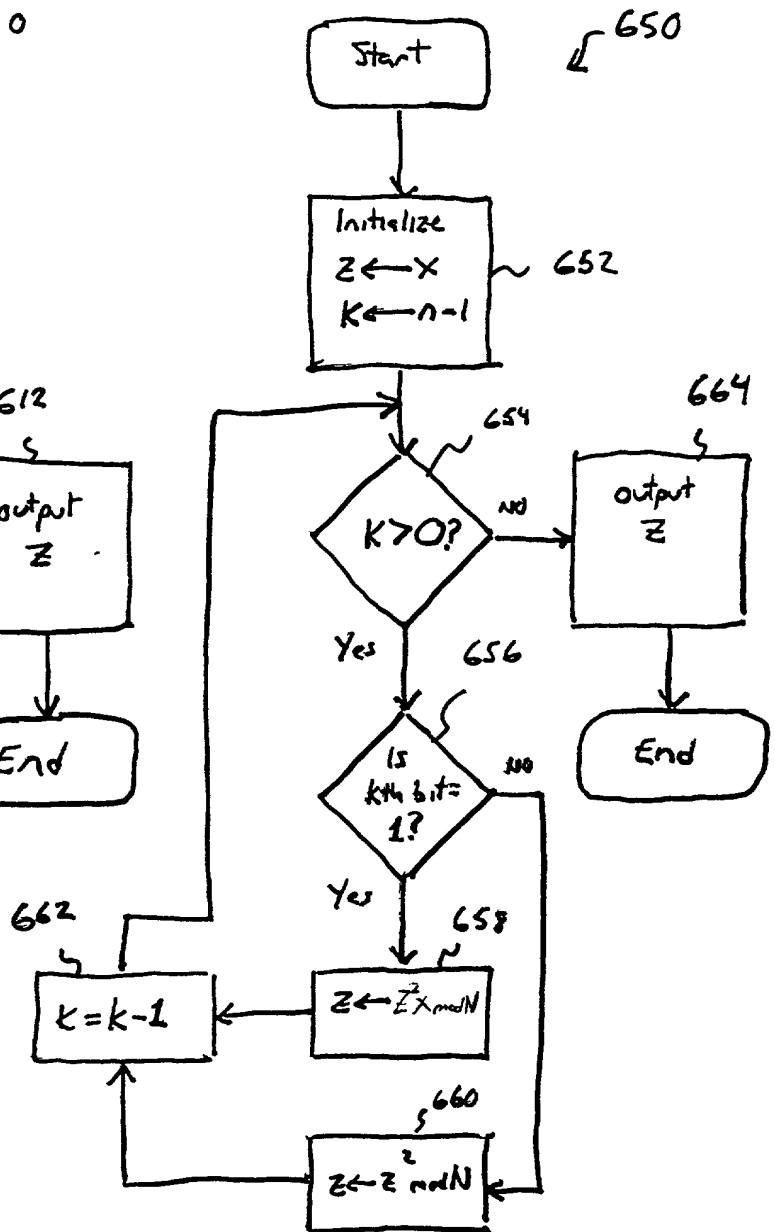


Fig. 7

